Measuring Business Cycles

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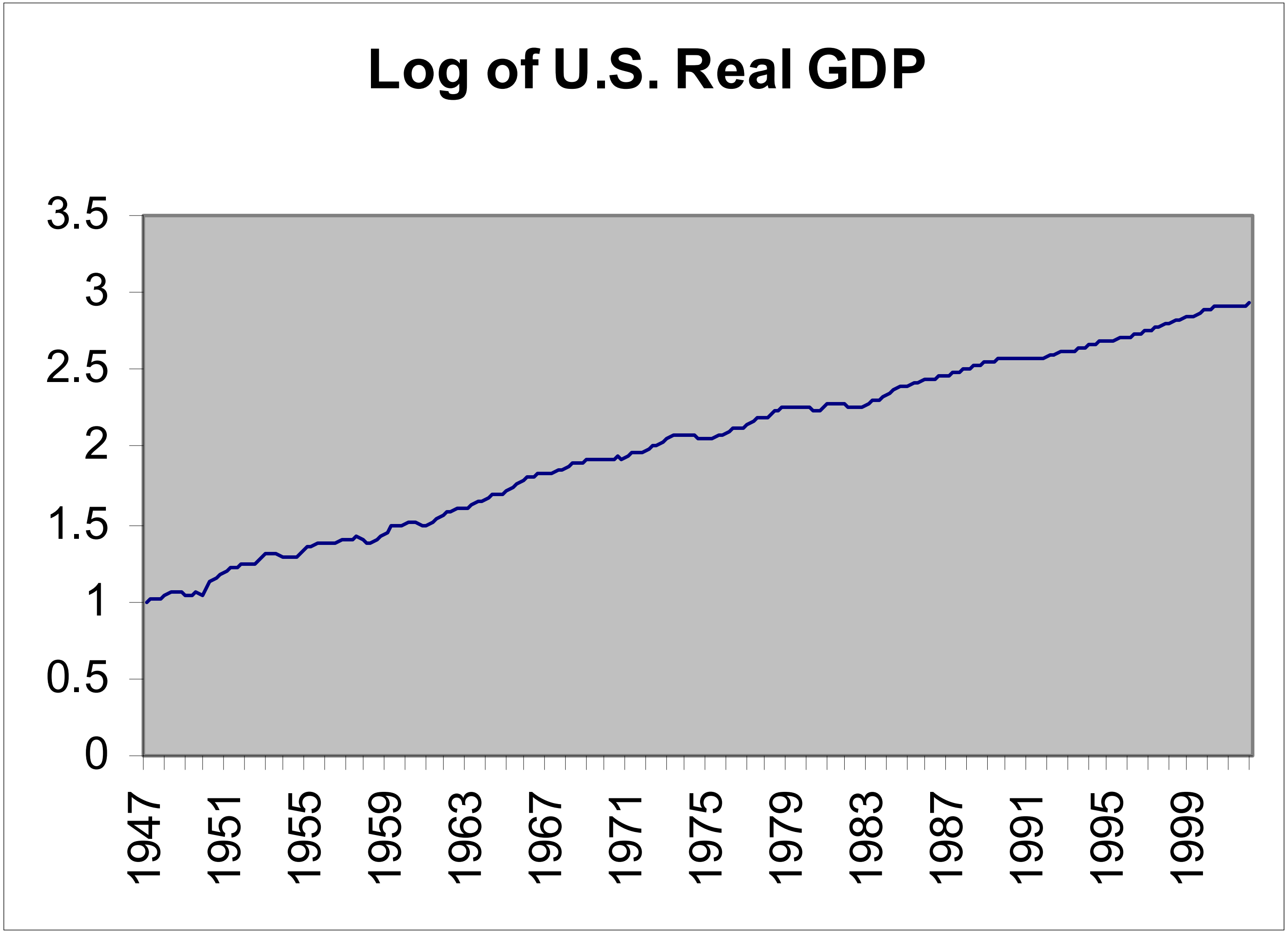
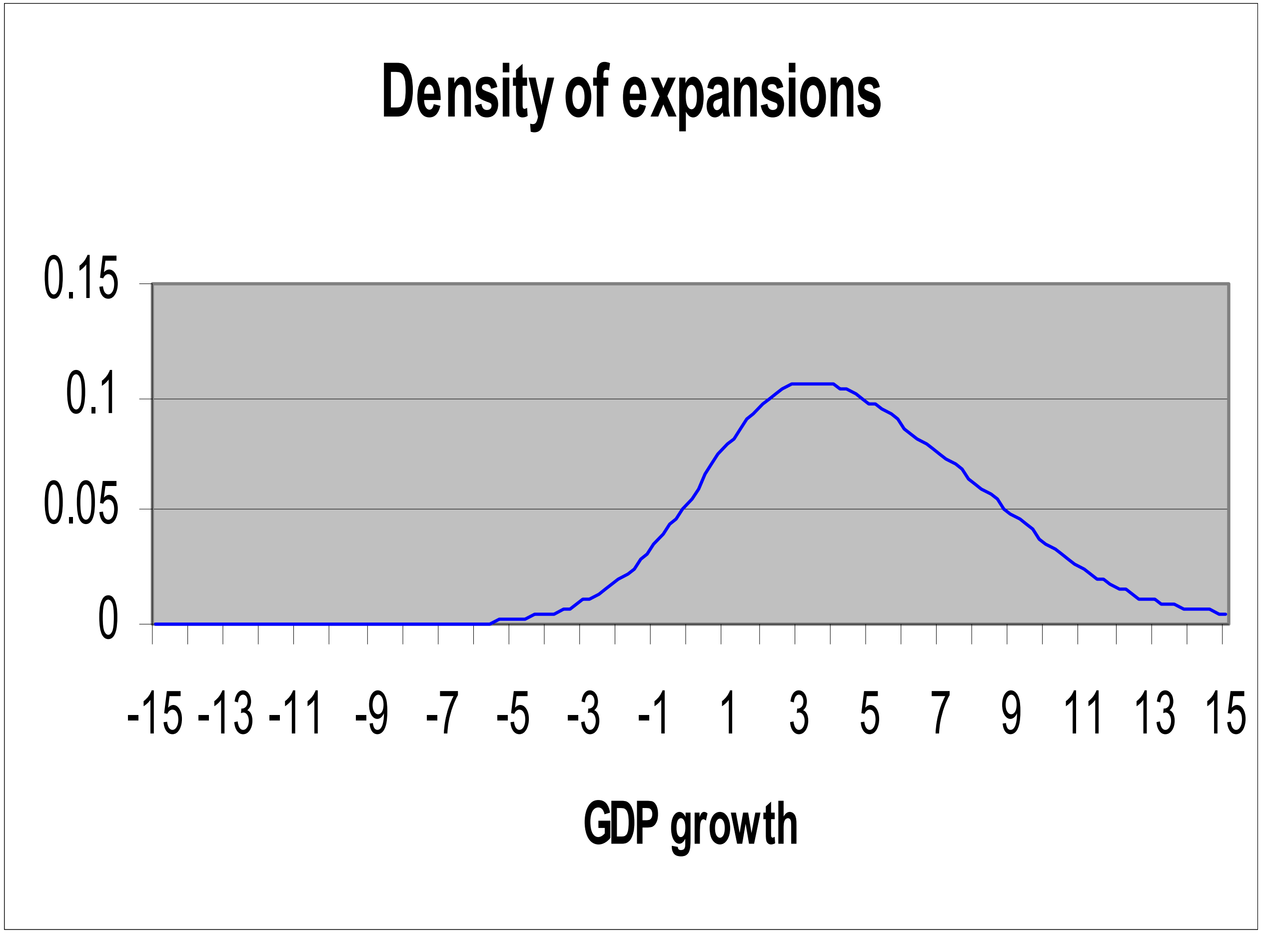
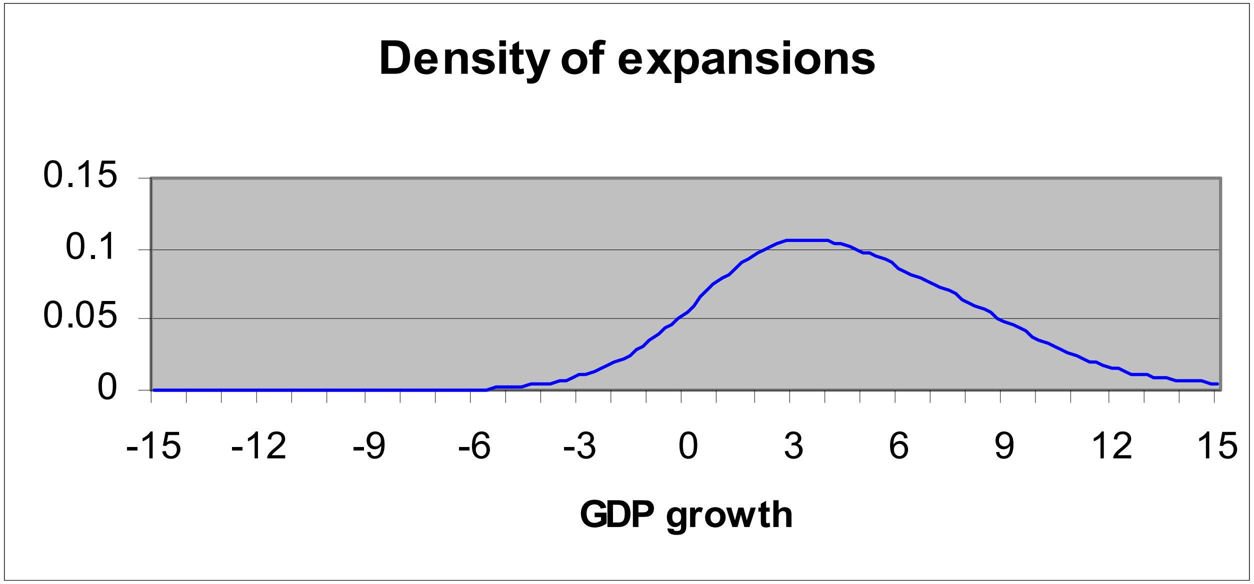

Log of U.S. Real GDP
3.5
3
2.5
2
1.5
1
0.5
0
1947
1951
1955
1959
1963
1967
1971
1975
1979
1983
1987
1991
1995
1999



$\Omega = 4.7$

$\alpha = 3.5$



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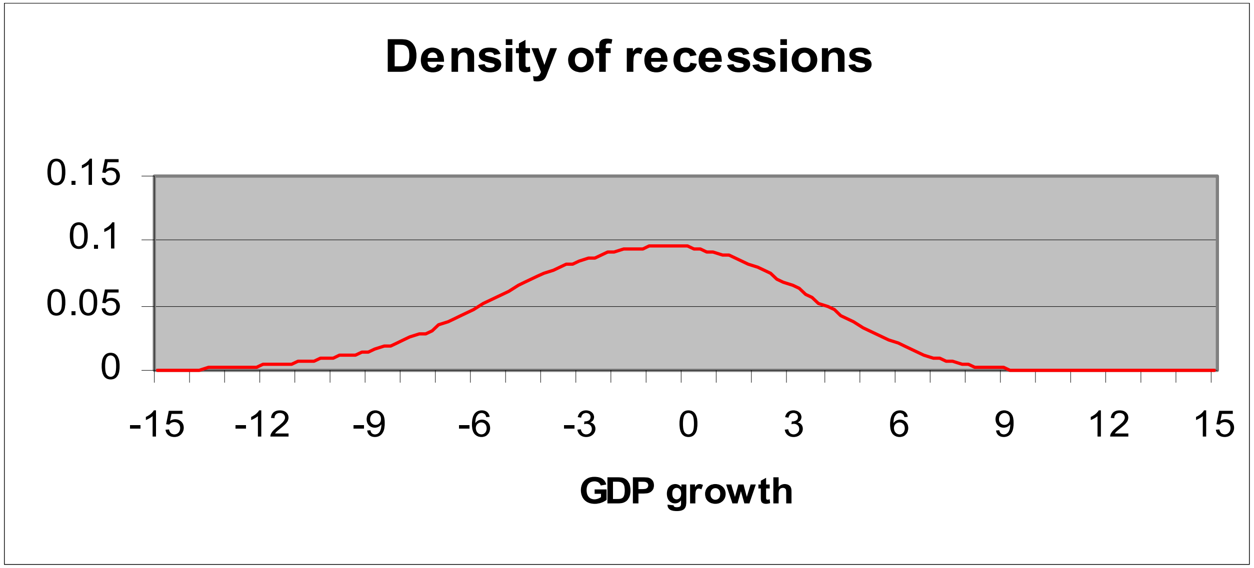
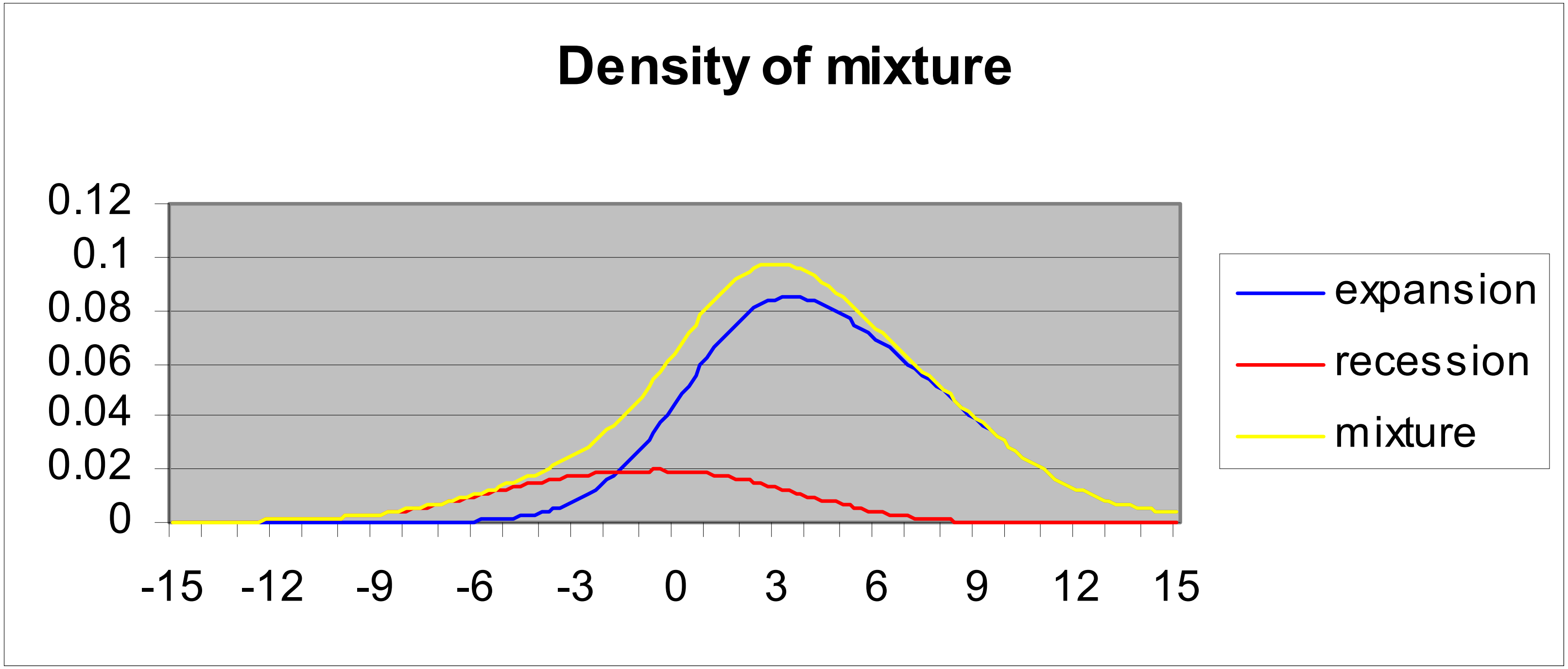
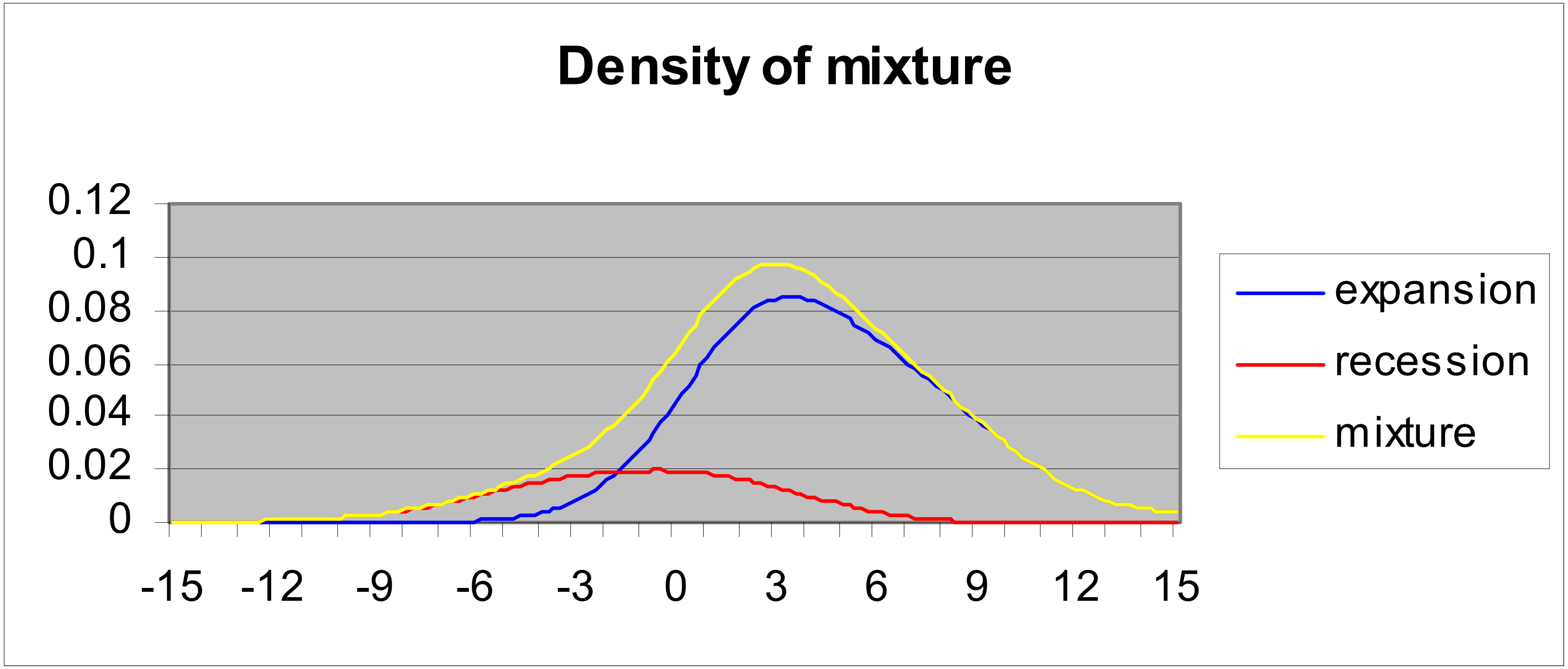


$\Omega = -1.2$
$\alpha = 3.5$

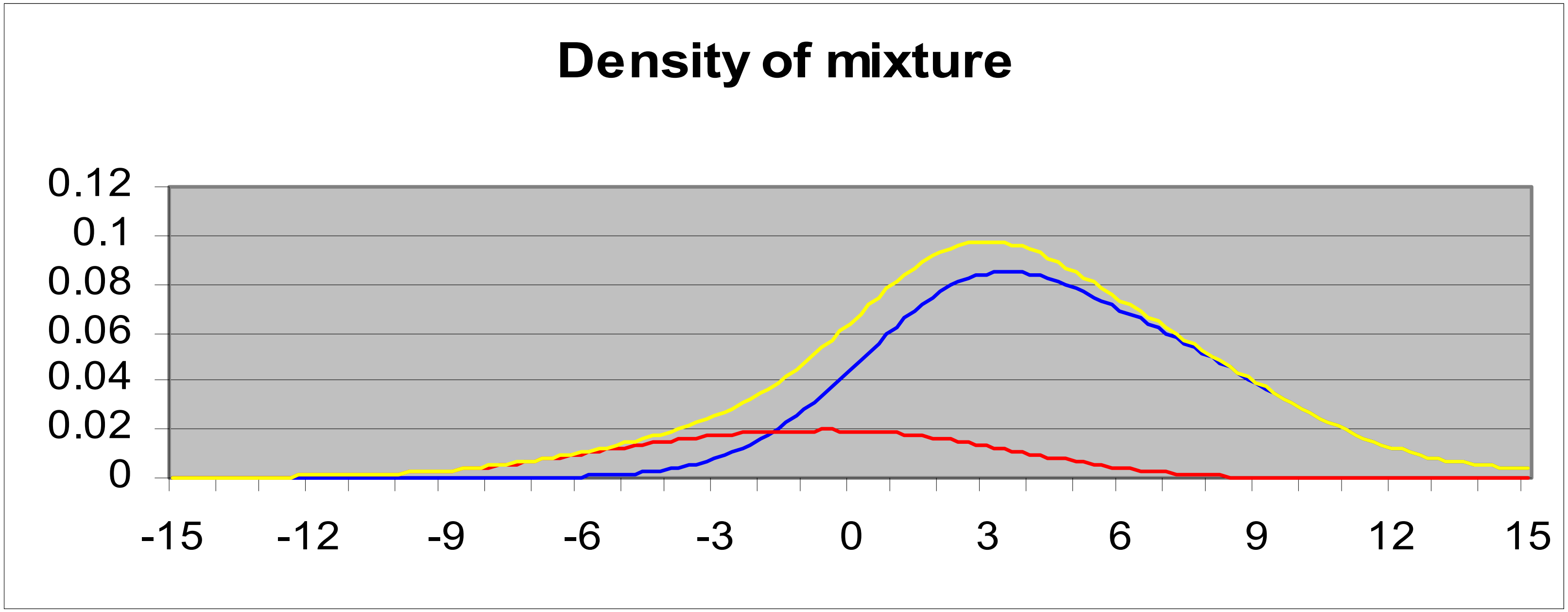

Density of mixture
0.12
0.1
0.08
0.06
0.04
0.02
0
-15
-12
-9
-6
-3
0
3
6
9
12
15
expansion
recession
mixture



$$\Pr(S_t = 2 \mid y_t) = \frac{\Pr(S_t = 2, y_t)}{f(y_t)}$$

$$= \frac{\Pr(S_t = 2, y_t)}{\Pr(S_t = 1, y_t) + \Pr(S_t = 2, y_t)}$$

$$\Pr(S_t = 2, y_t) = \Pr(S_t = 2) \cdot f(y_t \mid S_t = 2)$$


Density of mixture
0.12
0.1
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-15
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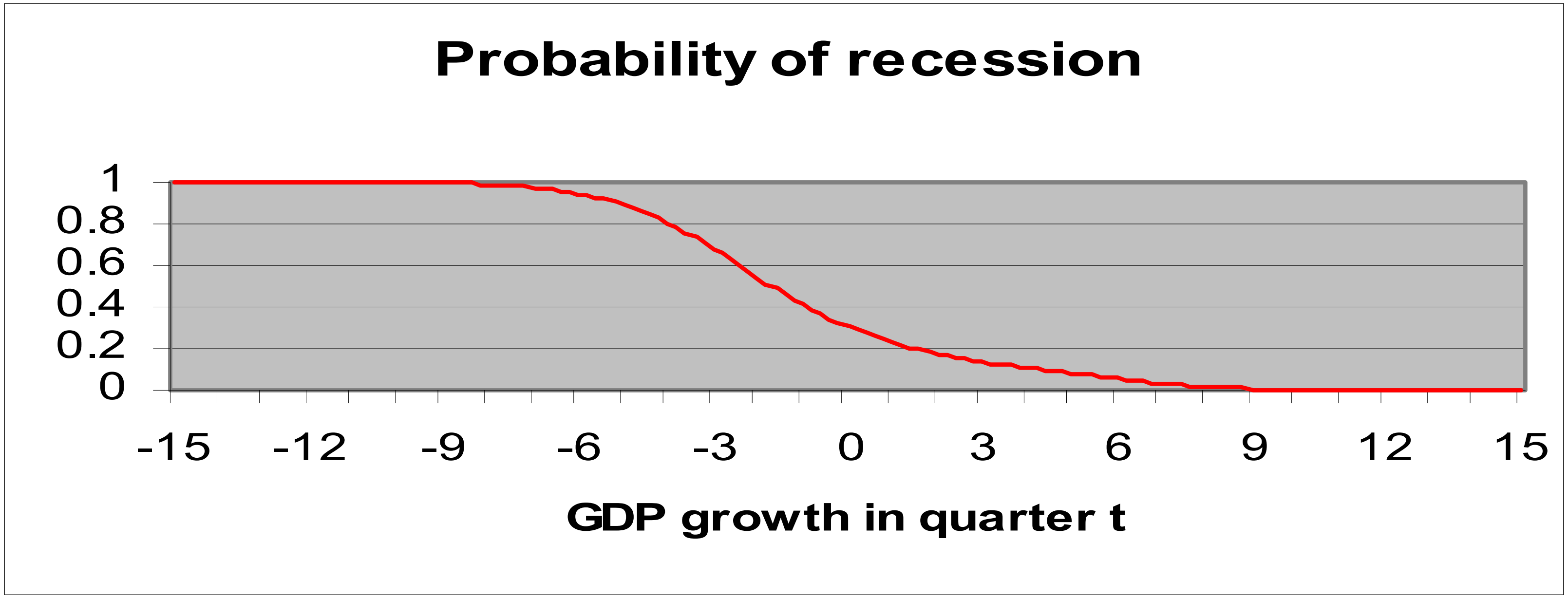
3
6
9
12
15


Probability of recession
1
0.8
0.6
0.4
0.2
0
-15
-12
-9
-6
-3
0
3
6
9
12
15
GDP growth in quarter t

Prob(NBER expansion in t + 1| NBER expansion in t)
= 164/174 = 0.94
Prob(NBER contraction in t +1 | NBER contraction in t)
= 35/44 = 0.80

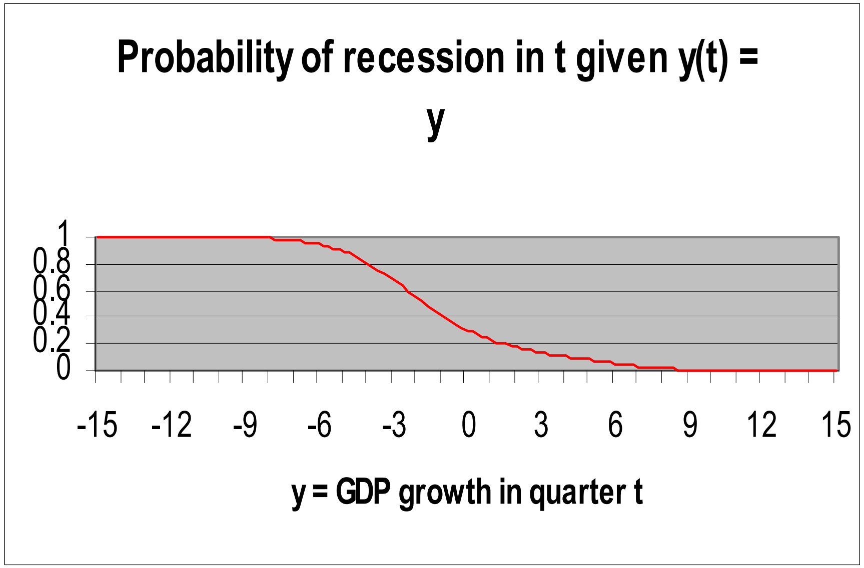
$$\Pr(S_{t+1} = 2 \mid y_{t+1}, y_t) = \frac{\Pr(S_{t+1} = 2, y_{t+1} \mid y_t)}{f(y_{t+1} \mid y_t)}$$

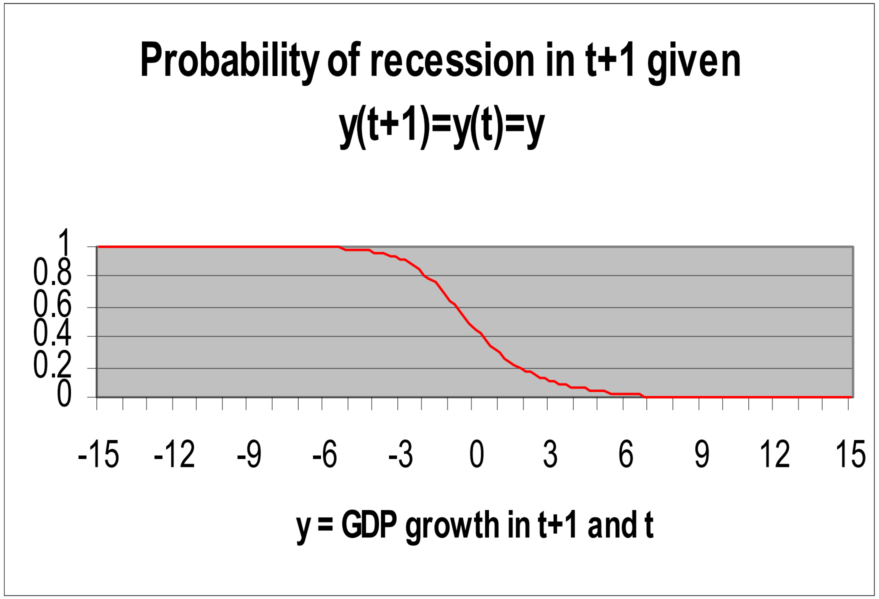
$$= \frac{\Pr(S_{t+1} = 2, y_{t+1} \mid y_t)}{\Pr(S_{t+1} = 1, y_{t+1} \mid y_t) + \Pr(S_{t+1} = 2, y_{t+1} \mid y_t)}$$

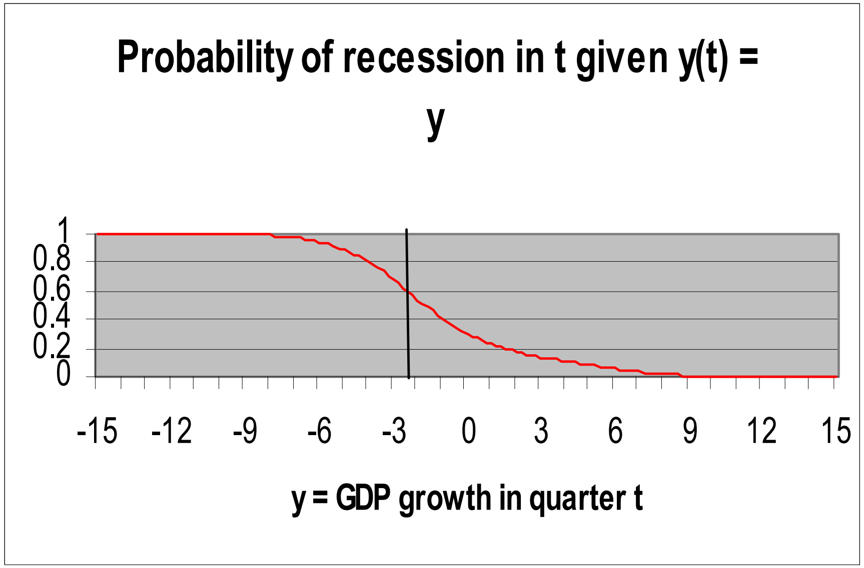
$$\Pr(S_{t+1} = 2, y_{t+1} \mid y_t) = \sum_{s=1}^{2} \Pr(S_{t+1} = 2, S_t = s, y_{t+1} \mid y_t)$$

$$\Pr(S_{t+1} = 2, S_t = s, y_{t+1} \mid y_t) =$$

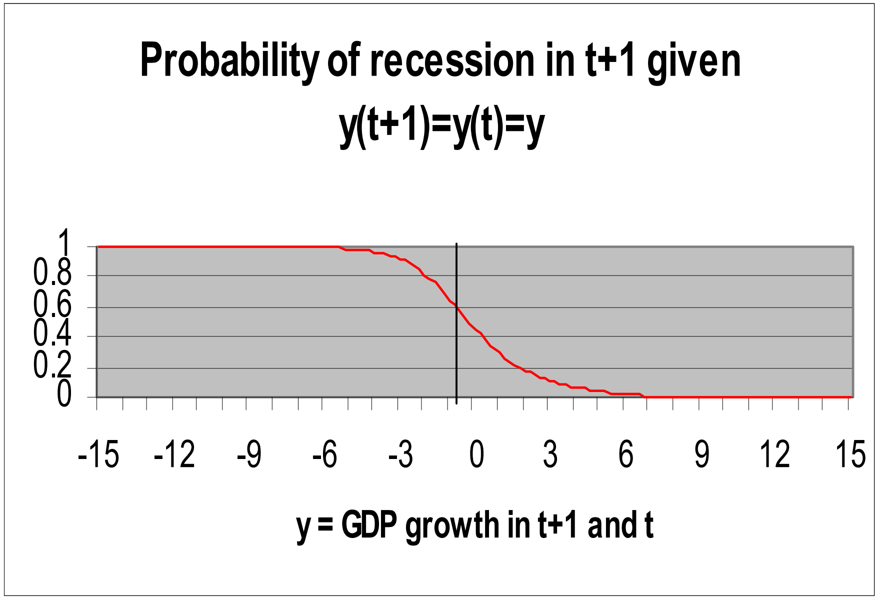
$$\underbrace{\Pr(S_t = s \mid y_t)}_{\substack{\text{known from} \\ \text{previous slide}}} \cdot \underbrace{\Pr(S_{t+1} = 2 \mid S_t = s, y_t)}_{\substack{0.80 \text{ for s} = 2 \\ 0.06 \text{ for s} = 1}} \cdot \underbrace{f(y_{t+1} \mid S_{t+1} = 2, S_t = s, y_t)}_{\substack{\text{known from} \\ \text{density of recessions}}}$$


Probability of recession in t given y(t) = y
1
0.8
0.6
0.4
0.2
0
-15
-12
-9
-6
-3
0
3
6
9
12
15
y = GDP growth in quarter t


Probability of recession in t+1 given y(t+1)=y(t)=y
1
0.8
0.6
0.4
0.2
0
-15
-12
-9
-6
-3
0
3
6
9
12
15
y = GDP growth in t+1 and t



Prob = 0.6
at y = -2.2



Prob = 0.6
At y = -0.8

Can also calculate “smoothed probability”

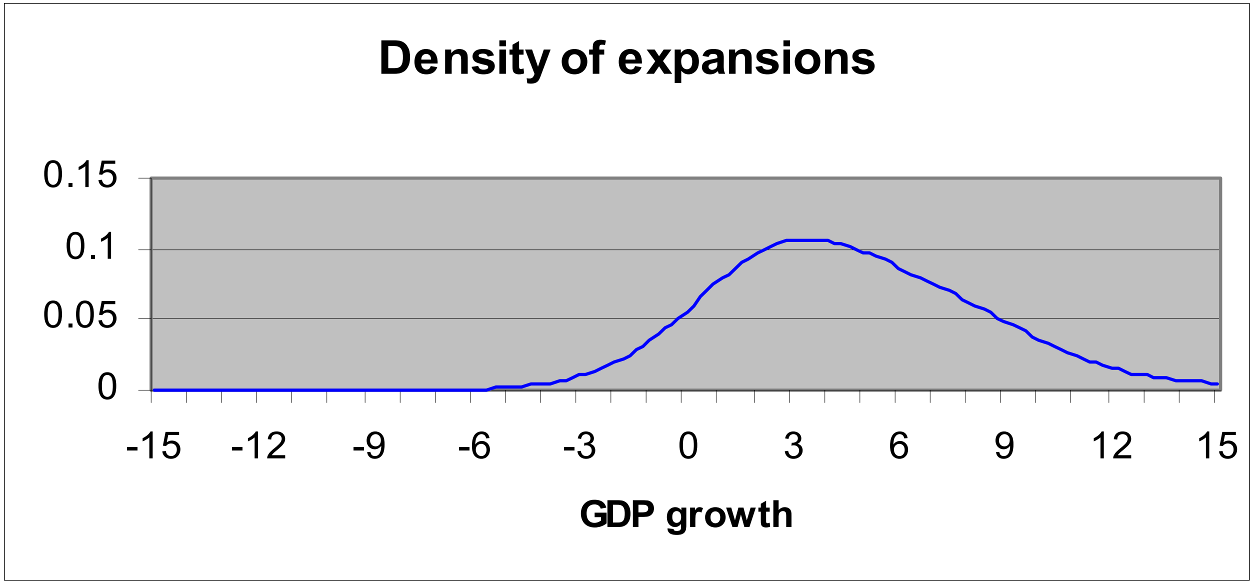
$$\Pr(S_t = 2 \mid y_t, y_{t+1})$$

Nonparametric summary of NBER labels:

$f(y_t \mid S_t = 1)$ estimated nonparametrically

$$\Pr(S_{t+1} = 1 \mid S_t = 1) = 0.94$$

$$\Pr(S_{t+1} = 2 \mid S_t = 2) = 0.80$$



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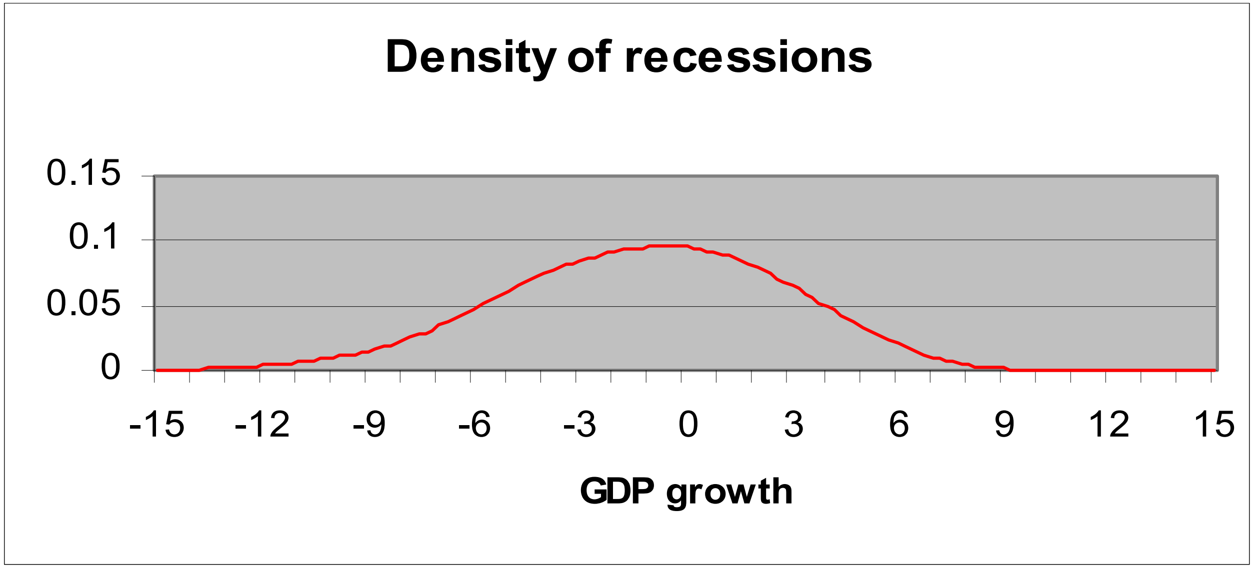

$\Omega = -1.2$
$\alpha = 3.5$

Could also approach parametrically:

$$f(y_t \mid S_t = 1) \sim N(\mu_1, \sigma^2)$$

$$f(y_t \mid S_t = 2) \sim N(\mu_2, \sigma^2)$$

$$\Pr(S_{t+1} = 1 \mid S_t = 1) = p_{11}$$

$$\Pr(S_{t+1} = 2 \mid S_t = 2) = p_{22}$$

$$\Pr(S_{t+1}=2, S_t=s, y_{t+1} \mid y_t) =$$

$$\underbrace{\Pr(S_t=s \mid y_t)}_{\substack{\text{known from}\\ \text{previous step}}} \cdot \underbrace{\Pr(S_{t+1}=2 \mid S_t=s, y_t)}_{\substack{\text{p22 for s = 2}\\ \text{p11 for s = 1}}} \cdot \underbrace{f(y_{t+1} \mid S_{t+1}=2, S_t=s, y_t)}_{N(\mu_2, \sigma^2)}$$

$$f(y_{t+1} \mid y_t) =$$
$$\Pr(S_{t+1}=1, S_t=1, y_{t+1} \mid y_t) +$$
$$\Pr(S_{t+1}=1, S_t=2, y_{t+1} \mid y_t) +$$
$$\Pr(S_{t+1}=2, S_t=1, y_{t+1} \mid y_t) +$$
$$\Pr(S_{t+1}=2, S_t=2, y_{t+1} \mid y_t)$$

Parameter	Estimated from NBER dates and GDP	Estimated from GDP alone
Ω_1	4.73	4.86
Ω_2	-1.19	-0.27
α	3.47	3.54
p_{11}	0.94	0.91
p_{22}	0.80	0.75

1990 recession ended:

1991:IV (full sample)
1992:I (current)
1991:II (NBER)

1990 recession ended:
1991:IV (full sample)
1992:I (current)
1991:II (NBER)

2001 recession ended:
2001:IV (full sample)
2002:I (current)