



# Income Misattribution under Formula Apportionment

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## How multinational firms are taxed.

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- The basic idea is that country A taxes a firm based on income earned in A, and country B taxes the firm based on income earned in B.
- We may (or may not!) know how much income the firm has earned this year, but to implement this taxation scheme it is necessary to identify the distribution of these earnings between A and B.
- The current system of international profit allocation: separate accounting with arm's length prices used to book intrafirm transactions.
- Hence the firm calculates separately its profits in A and its profits in B, and is not allowed to use trade between A and B at fictitious prices to massage taxable income out of high-tax jurisdictions and into low-tax jurisdictions.
- U.S. states, however, use formulary methods...

## Formulary methods used by U.S. states.

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- Multistate businesses do not keep separate accounts for their operations in each state.
- Instead they determine their Ohio profits (for purpose of Ohio taxation) by taking total U.S. profits, and determining Ohio's share based on:
  - Employment
  - Sales
  - Tangible property
- To make it even more fun, states use formulas with differing weights on these three factors (e.g., Ohio is 60-20-20, whereas Oklahoma is 33-33-33).
- What's up with the formulas? State tax rates are low, and it's too much of a hassle to determine state profits exactly. The formula factors may be convenient proxies.

# Background

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- There is quite a bit of interest in formulary alternatives to separate accounting, reflecting...
  - Concerns about tax avoidance
  - Concerns about various forms of tax competition (statutory rates, bases, apportionment weights)
  - General desires for unity and harmonization.
- There is a clumsy aspect to formula apportionment, in that no one actually thinks that the factors used in these formulas in fact capture income production. The question is, how bad is that? (And if it's bad, which factors are the worst?)
- Formulas distinguish big operations from small operations, and on average, big operations produce large incomes.



# Ownership

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- Some of the difficulties associated with formulary apportionment stem from the fact that formulary methods apply to controlled groups.
  - This is sensible if the idea is to reduce incentives to reallocate taxable income from one jurisdiction to another.
  - The problem is that the ownership-based inclusion criterion then creates its own incentives for ownership.
  - Note: in the U.S. state case, common ownership is not sufficient; a business must also be unitary.
- Ownership implications: incentives for mergers and asset divestment.

## More on mergers and divestitures.

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- Consider an extreme example.
  - A firm with high profits in Germany (high tax).
  - Acquires an unprofitable firm with lots of employment in Ireland (low tax).
  - With separate accounting the firm's profits are taxed in Germany.
  - With formulary apportionment (based on employment) the firm's total taxes are reduced. Hence formulary apportionment creates a merger incentive.
- Gordon and Wilson (1986) note that, with zero merger costs, there are so many mergers that all firms wind up with the same tax rates.
- But: what if mergers and divestitures are costly?
  - We get incomplete tax arbitrage, and...
  - The induced ownership changes are associated with economic inefficiency.

# What we know about mergers & other determinants of asset ownership.

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- There is a sizeable literature on tax effects of mergers and divestments. Bottom line: there are significant tax effects.
- There is also a sizeable literature on the effects of taxation on FDI (and most FDI is acquisition-based), likewise finding significant tax effects.
- We also know that mergers are costly; we know that because many firms could improve their tax positions (e.g., take advantage of unused tax loss carryforwards) by merging with other firms, and the evidence is that many leave these tax benefits on the table by not merging.

## What this paper does.

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- The paper looks at data on separate companies to estimate the extent to which formulary factors (sales, property, employment) explain variation in reported profitability.
- Why do this?
- The paper shows that we can use a regression to estimate coefficients on various factors explaining profitability, and that these coefficients are the ownership distortion-minimizing factor weights.
- This offers a theoretical justification for a procedure in that is (relatively) easy to implement and produces answers that are easy to interpret.
- The point is to think of the formulas as predictions, and to ask how good these predictions are, and what kind of distortions are introduced by having (inevitably) bad predictions.

## The evidence.

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- The paper uses Amadeus data for European firms in 2004 (results for American firms are similar).
- The results indicate that observable factors do a poor job of predicting profits.
  - Property, employment and sales explain less than 22% of the variation in profits among European firms.
  - Absolute value of European prediction errors exceeds half of predicted profits 64% of the time; they exceed twice predicted profits 11% of the time.
- Employment is by far the worst factor.
  - Estimated effect is typically insignificant or negative (particularly in the American data).
  - This may not be surprising, given that labor costs are deductible.

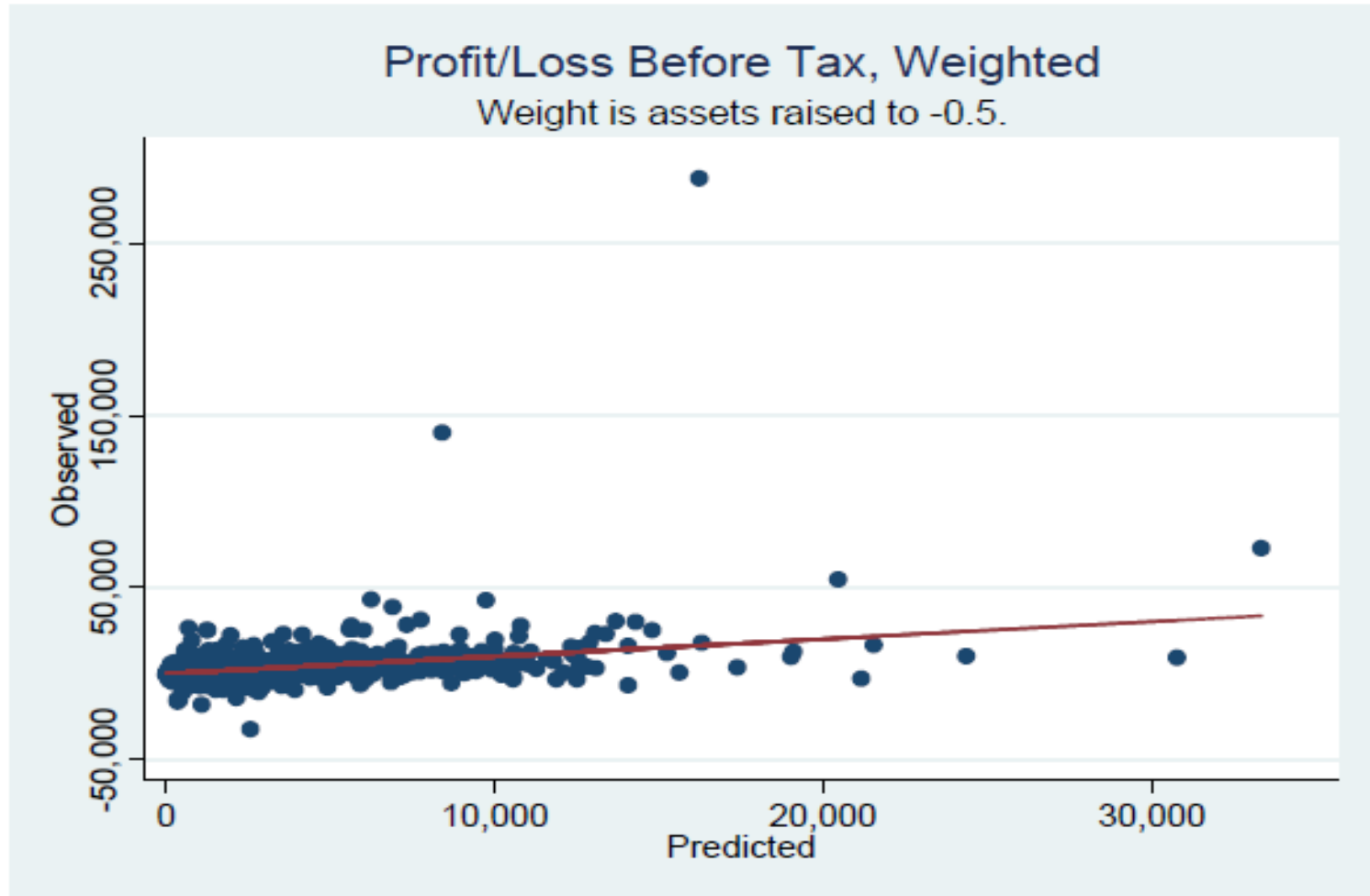
## European Data, European Evidence.

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- Consolidated accounting information for 2004, firms with 100 or more employees in 34 European countries.
- Income variables:
  - Profit/loss before tax.
  - Enterprise value (yearend market capitalization).
- Formulary factors:
  - Total worldwide sales.
  - Book value of property, plant and equipment.
  - Number of employees.
  - Labor compensation.
- Similar results for 5-year averages, other years.
- The pictures tell the story: sales, property, and employment poorly predict income or market capitalization.

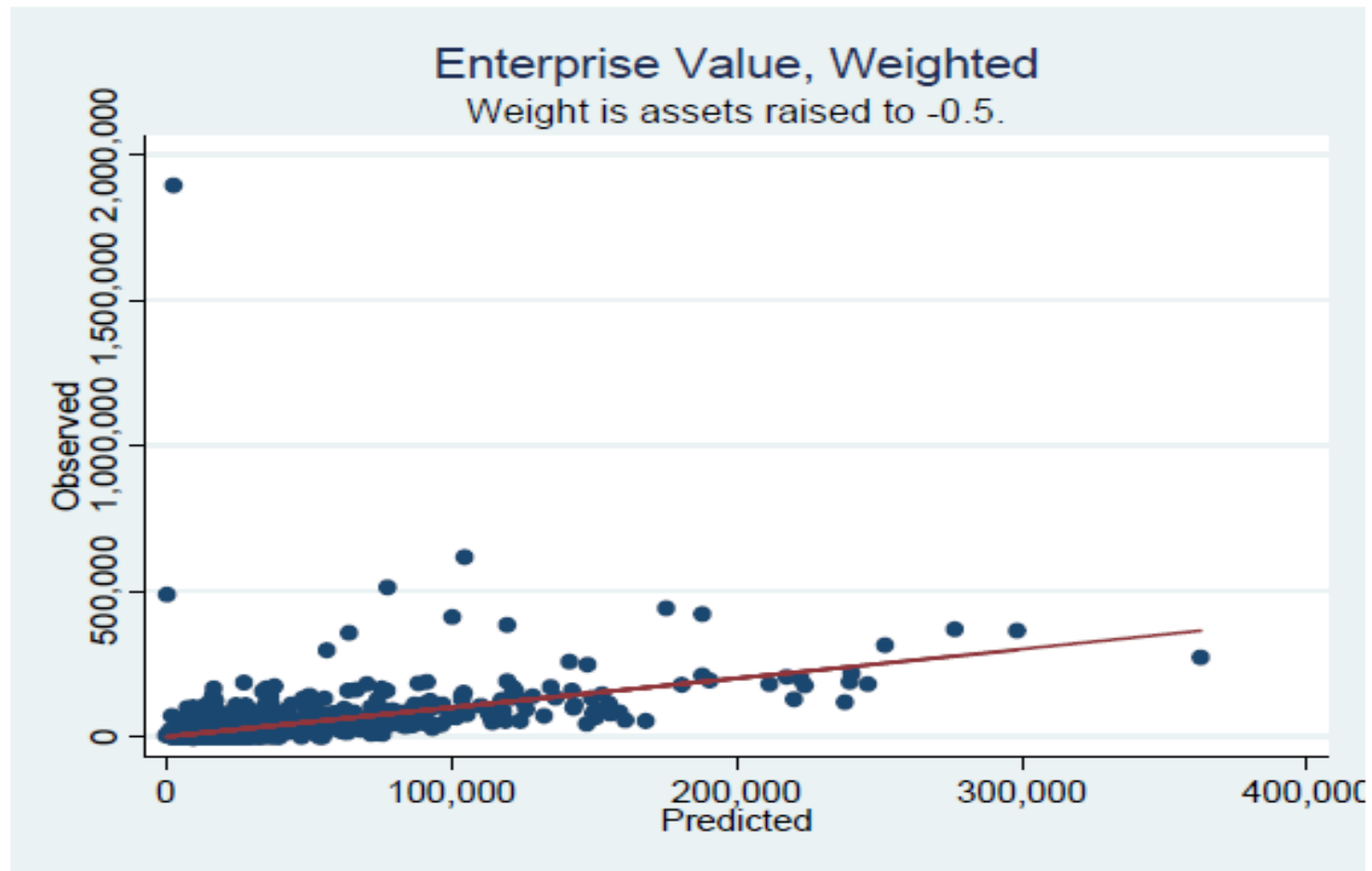
**Figure 1**

**Errors in Unconstrained Equations Predicting Profits and Losses**



**Figure 2**

**Errors in Unconstrained Equations Predicting Enterprise Market Value**



**Table 2**  
**Determinants of Profits, 2004, WLS Results**

Notes: The Table presents estimated coefficients from regressions explaining 2004 profits as functions of contemporaneous sales, capital, and labor inputs. All equations are estimated using weighted least squares, with weights equal to one divided by the square root of contemporaneous firm assets. Robust standard errors are presented in parentheses.

<i>Dependent Variable:</i>	Profit/Loss Before Tax		Enterprise Value	
	(1)		(2)	
Sales	0.6095 (0.1462)	0.5908 (0.1099)	0.2480 (0.0552)	0.2061 (0.0927)
Property, Plant and Equipment	0.2560 (0.0832)	0.2599 (0.0814)	0.4754 (0.0368)	0.4991 (0.0407)
Labor Compensation	0.0654 (0.0655)		0.1532 (0.0478)	
Employment		0.0784 (0.0721)		0.1627 (0.0701)
No. of Obs.	11,103	11,103	1,473	1,473
R-Squared	0.2109	0.2135	0.3404	0.3629

**Table 3**  
**Determinants of Profits, Two-Factor, WLS Results**

Notes: The Table presents estimated coefficients from regressions explaining 2004 profits as functions of contemporaneous sales and labor inputs. All equations are estimated using weighted least squares, with weights equal to one divided by the square root of contemporaneous firm assets. Robust standard errors are presented in parentheses.

<i>Dependent Variable:</i>	Profit/Loss Before Tax		Enterprise Value	
	(1)		(2)	
Sales	0.7158 (0.1331)	0.7227 (0.0912)	0.5987 (0.0702)	0.5688 (0.0931)
Labor Compensation	0.1000 (0.0657)		0.1200 (0.0582)	
Employment		0.0836 (0.0716)		0.1366 (0.0771)
No. of Obs.	11,103	11,103	1,473	1,473
R-Squared	0.1996	0.2017	0.2787	0.2953

**Table 5: Prediction Errors in the Unconstrained - Three Factor Model**

Model	Profit/Loss Before Tax		Enterprise Value	
	1	2	1	2
Number of Firms	11,103	11,103	1,473	1,473
mean[ $ \pi\text{-hat}(\pi)  / \text{hat}(\pi)$ ]	1.09	1.08	2.43	1.00
median[ $ \pi\text{-hat}(\pi)  / \text{hat}(\pi)$ ]	0.69	0.69	0.50	0.51
Percent of firms for which:				
$ \pi\text{-hat}(\pi)  > 0.1 * \text{hat}(\pi)$	93.11%	92.88%	90.77%	90.97%
$ \pi\text{-hat}(\pi)  > 0.25 * \text{hat}(\pi)$	82.28%	82.37%	75.56%	76.37%
$ \pi\text{-hat}(\pi)  > 0.5 * \text{hat}(\pi)$	63.69%	63.95%	49.63%	51.60%
$ \pi\text{-hat}(\pi)  > 1 * \text{hat}(\pi)$	30.42%	30.38%	14.80%	16.23%
$ \pi\text{-hat}(\pi)  > 2 * \text{hat}(\pi)$	11.02%	10.83%	8.35%	9.23%
$ \pi\text{-hat}(\pi)  > 5 * \text{hat}(\pi)$	2.04%	1.95%	2.58%	3.26%

**Notes:**

Model 1 is the following specification: dependent variable regressed on sales, net PPE, and labor costs. Model 2 is identical to Model 1 except that number of employees is substituted for labor costs.

A firm's actual profit level or enterprise value is denoted  $\pi$ , whereas the predicted value is denoted  $\text{hat}(\pi)$ .

## Evidence from American firms.

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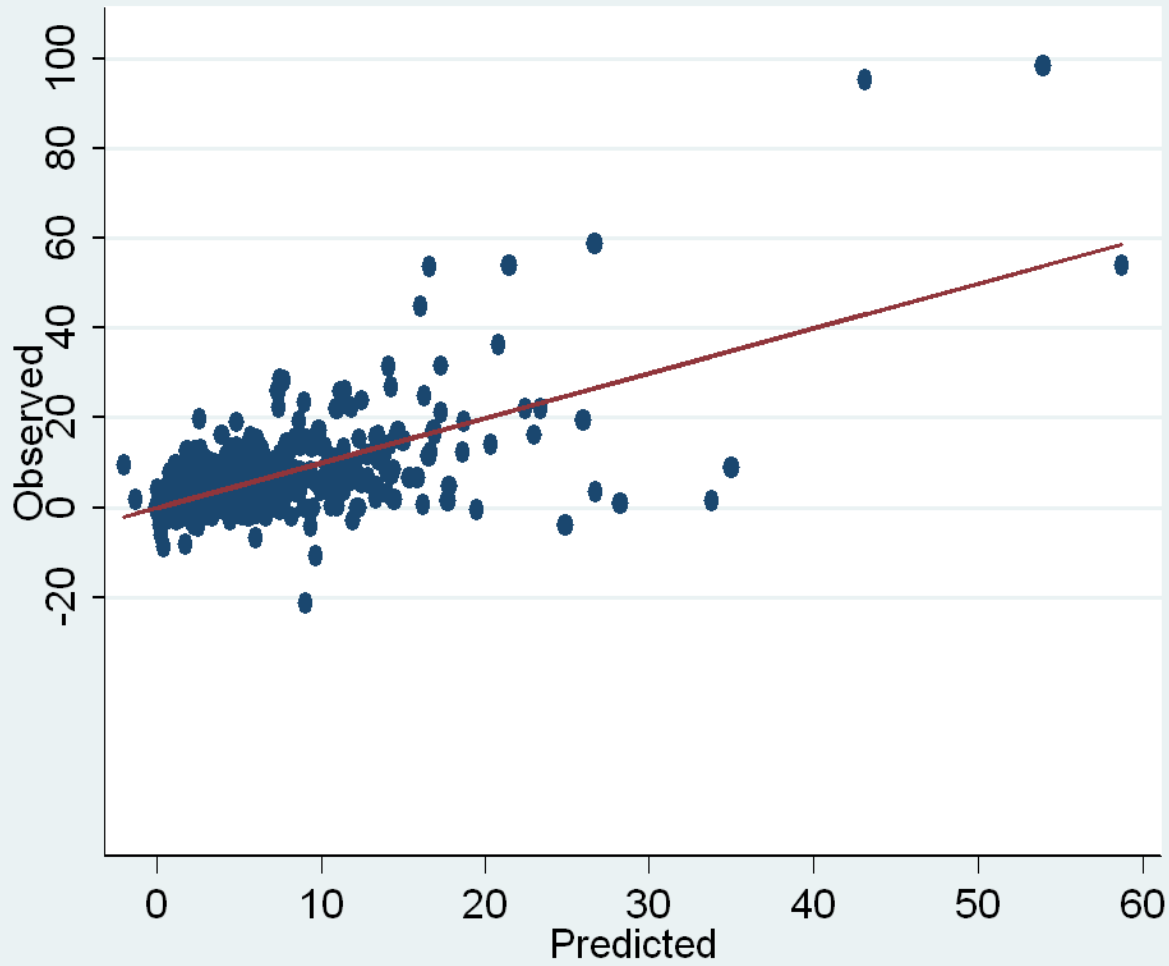
- Additional regressions (not reported in the circulated paper) indicate that observable factors such as sales, property, and employment do an even worse job explaining the profits of American firms than they do explaining the profits of European firms.
- How well do they do?
  - Property, employment and sales explain less than 58% of the variation in profits among American firms.
  - There are many large unexplained profits: the absolute value of prediction errors exceeds predicted profits 53% of the time, and exceeds three times predicted profits 29% of the time.
- Employment is by far the worst factor.
  - The estimated employment effect is often negative.

# American Data, American Evidence.

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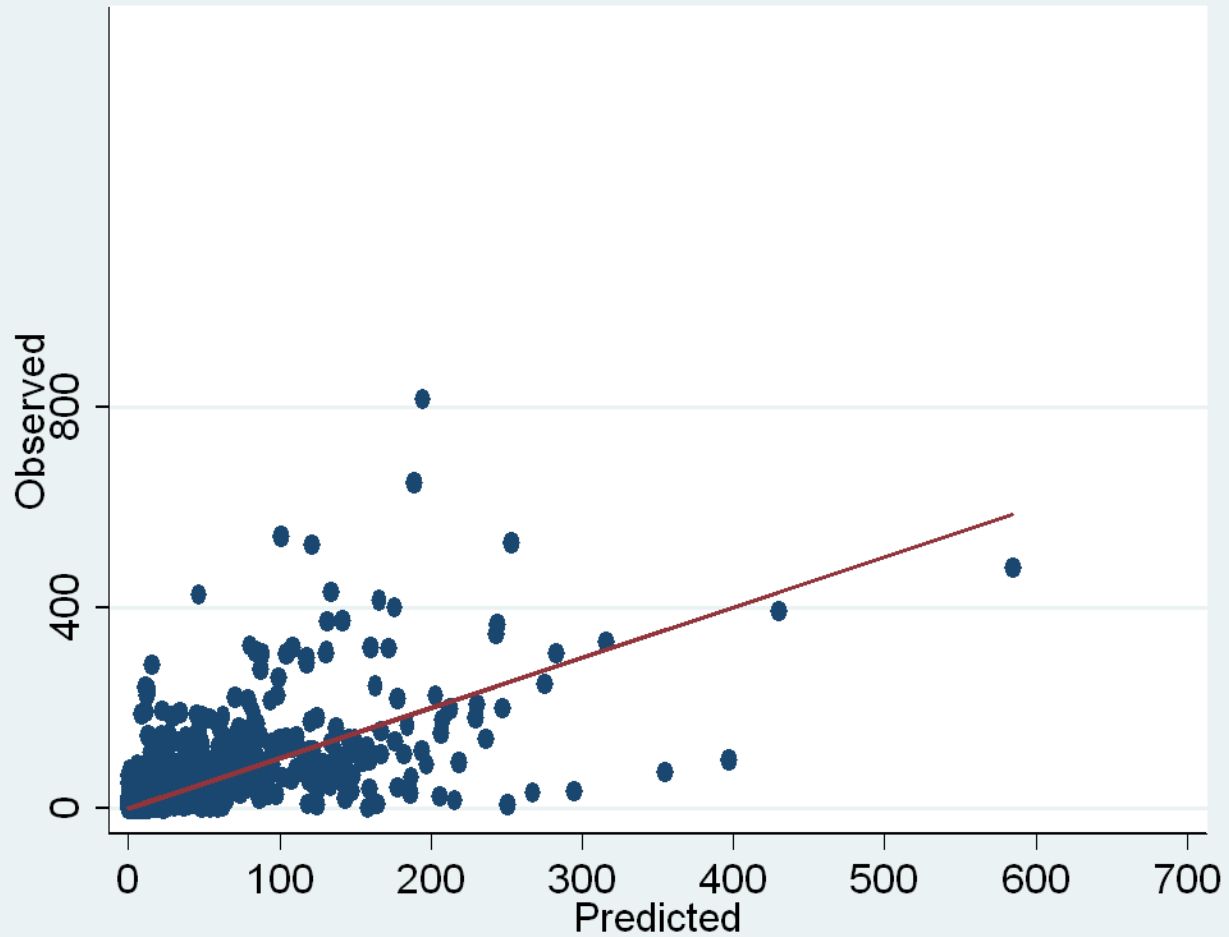
- Compustat, 2004
- Income variables:
  - Taxes paid/0.35.
  - Operating income after depreciation.
  - Yearend market capitalization.
- Formulary factors:
  - Total worldwide sales.
  - Book value of property, plant and equipment.
  - Number of employees (almost 7,000 observations).
  - Employment compensation (only about 1,500 observations).
- Similar results for 5-year averages, other years.
- Again, the pictures tell the story:

come Derived from Taxes Paid, Weighted: sam  
Weight is assets raised to -0.5.



# Market Capitalization, Weighted: sample 1

Weight is assets raised to -0.5.



**Table 2**  
**Determinants of Profits, 2004, WLS Results**

Notes: The Table presents estimated coefficients from regressions explaining 2004 profits as functions of contemporaneous sales, capital, and labor inputs. All equations are estimated using weighted least squares, with weights equal to one divided by the square root of contemporaneous firm assets. Robust standard errors are presented in parentheses.

<i>Dependent Variable:</i>	Operating Income		Taxable Income as Reflected in Taxes Paid		Market Capitalization	
	(1)	(2)	(3)	(4)	(5)	(6)
Sales	0.4472 (0.0638)	0.4388 (0.0291)	0.7214 (0.1439)	0.5886 (0.0518)	0.5076 (0.0877)	0.5597 (0.0410)
Property, Plant and Equipment	0.1592 (0.0406)	0.2862 (0.0208)	0.1554 (0.0617)	0.2826 (0.0343)	0.1786 (0.0462)	0.2108 (0.0259)
Labor Compensation	0.0072 (0.0159)		-0.0349 (0.0415)		0.0948 (0.0329)	
Employment		-0.0015 (0.0027)		-0.0065 (0.0054)		0.0029 (0.0028)
No. of Obs.	1,575	6,992	1,574	6,986	1,461	6,384
R-Squared	0.5425	0.5427	0.5715	0.5004	0.5281	0.4049

**Table 4**  
**Determinants of Profits, Two-Factor, WLS Results**

Notes: The Table presents estimated coefficients from regressions explaining 2004 profits as functions of contemporaneous sales and labor inputs. All equations are estimated using weighted least squares, with weights equal to one divided by the square root of contemporaneous firm assets. Robust standard errors are presented in parentheses.

<i>Dependent Variable:</i>	Operating Income		Taxable Income as Reflected in Taxes Paid		Market Capitalization	
	(1)	(2)	(3)	(4)	(5)	(6)
Sales	0.5938 (0.0535)	0.6343 (0.0292)	0.8645 (0.1182)	0.7816 (0.0536)	0.6750 (0.0699)	0.7086 (0.0339)
Labor Compensation	0.0088 (0.0171)		-0.0333 (0.0412)		0.0956 (0.0349)	
Employment		-0.0021 (0.0033)		-0.0070 (0.0059)		0.0027 (0.0031)
No. of Obs.	1,575	6,992	1,574	6,986	1,461	6,384
R-Squared	0.5174	0.4803	0.5581	0.4615	0.5083	0.3835

**Table 8: Unconstrained - Three Factor**

	<i>Operating Income</i>		<i>Taxable Income Derived from Taxes</i>		<i>Market Capitalization</i>	
	<b>1</b>	<b>2</b>	<b>1</b>	<b>2</b>	<b>1</b>	<b>2</b>
Sample <sup>1</sup>						
N (Sample) <sup>2</sup>	1575	6992	1574	6986	1461	6384
N (0-value for $\hat{\pi}$ ) <sup>3, 4</sup>	3	24	3	24	2	21
N (mean computation) <sup>5</sup>	1572	6968	1571	6962	1459	6363
mean[ $ \hat{\pi} - \pi  / \hat{\pi}$ ]	56.43	7822.66	2.28	-25.16	10.60	150.95
median[ $ \hat{\pi} - \pi  / \hat{\pi}$ ]	2.08	1.11	1.00	1.00	1.51	0.98
Percent of firms for which:						
$ \hat{\pi} - \pi  > 0.05 * \hat{\pi}$	98.47%	96.96%	97.77%	97.67%	98.42%	97.16%
$ \hat{\pi} - \pi  > 0.1 * \hat{\pi}$	96.25%	94.00%	95.86%	95.65%	96.44%	94.06%
$ \hat{\pi} - \pi  > 0.2 * \hat{\pi}$	92.88%	88.09%	91.79%	91.12%	92.53%	88.70%
$ \hat{\pi} - \pi  > 0.25 * \hat{\pi}$	90.39%	85.40%	89.88%	88.75%	90.13%	85.62%
$ \hat{\pi} - \pi  > 0.5 * \hat{\pi}$	82.06%	71.48%	79.95%	77.82%	80.40%	72.31%
$ \hat{\pi} - \pi  > 0.75 * \hat{\pi}$	74.43%	60.07%	70.02%	66.83%	71.01%	59.42%
$ \hat{\pi} - \pi  > 1 * \hat{\pi}$	68.64%	53.07%	46.98%	32.28%	61.48%	49.17%
$ \hat{\pi} - \pi  > 2 * \hat{\pi}$	51.46%	37.17%	24.76%	15.66%	38.93%	35.90%
$ \hat{\pi} - \pi  > 3 * \hat{\pi}$	31.81%	29.16%	12.60%	9.41%	22.76%	27.27%
$ \hat{\pi} - \pi  > 4 * \hat{\pi}$	18.00%	23.13%	7.70%	5.90%	14.32%	21.91%
$ \hat{\pi} - \pi  > 5 * \hat{\pi}$	12.21%	19.25%	4.26%	4.45%	10.90%	18.76%
$ \hat{\pi} - \pi  > 7.5 * \hat{\pi}$	8.08%	14.85%	2.23%	2.47%	6.99%	14.25%
$ \hat{\pi} - \pi  > 10 * \hat{\pi}$	7.25%	12.94%	2.04%	1.94%	5.48%	11.99%

## What about simple formulas?

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- The evidence suggests that the formulas that most accurately capture income production would very heavily weight sales, and put small weights on property, and perhaps not weight employment at all.
- This is not what most states do (though sales weights are rising in popularity, with 11 states now apportioning income entirely according to sales). How far off does a state get by apportioning income using the Massachusetts formula (equal weights for sales, property, and employment) or double-weighting sales?
- Of course it is worse to constrain the weights in this manner.
- The evidence says that one increases by 25% the fraction of firms with prediction errors exceeding predicted profits if using the Massachusetts formula in place of optimal weights; and that this drops to 17.5% if using double-weighted sales.



## More on state formulas.

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- Using sales-only apportionment reduces the accuracy of predicted income by only 5.4% compared to using the optimal formulary weights.
- This reflects that the optimal weights are primarily piled on sales, and suggests that the movement in recent years of states to rely more heavily on sales apportionment may in fact more accurately reflect income production.
- Using equal-weighted three-factor formulas reduces the accuracy of predicted income by 19.4% compared to an optimal formula.
- Double-weighting sales reduces the accuracy of predicted income by 11% compared to the optimal formula.
- And relying entirely on employment weights reduces the accuracy of predicted income by 83%.

**Table 9: Constrained - Three Factor: (1/3 sales, 1/3 PPE, 1/3 labor measure)**

Sample <sup>1</sup>	Operating Income		Taxable Income Derived from Taxes		Market Capitalization	
	1	2	1	2	1	2
N (Sample) <sup>2</sup>	1575	6992	1574	6986	1461	6384
N (0-value for $\hat{\pi}$ ) <sup>3, 4</sup>	3	24	3	24	2	21
N (mean computation) <sup>5</sup>	1572	6968	1571	6962	1459	6363
mean[ $ \hat{\pi} - \pi  / \hat{\pi}$ ]	38.66	29.74	1.86	6.44	11.56	56.64
median[ $ \hat{\pi} - \pi  / \hat{\pi}$ ]	2.24	3.51	1.00	1.12	1.68	4.01
Percent of firms for which:						
$ \hat{\pi} - \pi  > 0.05 * \hat{\pi}$	98.22%	98.84%	97.64%	98.88%	98.29%	98.52%
$ \hat{\pi} - \pi  > 0.1 * \hat{\pi}$	95.99%	97.69%	96.12%	97.85%	96.71%	97.27%
$ \hat{\pi} - \pi  > 0.2 * \hat{\pi}$	93.07%	95.09%	92.11%	95.50%	92.67%	94.80%
$ \hat{\pi} - \pi  > 0.25 * \hat{\pi}$	91.16%	94.19%	90.52%	94.30%	90.68%	93.56%
$ \hat{\pi} - \pi  > 0.5 * \hat{\pi}$	83.84%	88.81%	81.54%	88.75%	81.15%	87.03%
$ \hat{\pi} - \pi  > 0.75 * \hat{\pi}$	76.59%	83.50%	72.37%	82.36%	71.90%	81.06%
$ \hat{\pi} - \pi  > 1 * \hat{\pi}$	70.61%	78.72%	50.03%	52.53%	63.47%	75.47%
$ \hat{\pi} - \pi  > 2 * \hat{\pi}$	54.45%	63.94%	29.34%	39.04%	44.21%	64.23%
$ \hat{\pi} - \pi  > 3 * \hat{\pi}$	36.64%	53.66%	16.49%	31.21%	27.48%	56.26%
$ \hat{\pi} - \pi  > 4 * \hat{\pi}$	22.52%	46.73%	9.23%	25.61%	17.82%	50.04%
$ \hat{\pi} - \pi  > 5 * \hat{\pi}$	16.35%	41.39%	6.37%	21.26%	12.41%	44.49%
$ \hat{\pi} - \pi  > 7.5 * \hat{\pi}$	9.67%	32.65%	2.55%	13.88%	7.61%	33.73%
$ \hat{\pi} - \pi  > 10 * \hat{\pi}$	7.44%	25.52%	1.21%	9.16%	5.41%	26.17%
vs. unconstrained model <sup>6</sup>						
<i>(percentage point difference)</i>						
$ \hat{\pi} - \pi  > 0.05 * \hat{\pi}$	-0.25%	1.88%	-0.13%	1.21%	-0.14%	1.37%
$ \hat{\pi} - \pi  > 0.1 * \hat{\pi}$	-0.25%	3.69%	0.25%	2.20%	0.27%	3.21%
$ \hat{\pi} - \pi  > 0.2 * \hat{\pi}$	0.19%	7.00%	0.32%	4.38%	0.14%	6.10%
$ \hat{\pi} - \pi  > 0.25 * \hat{\pi}$	0.76%	8.78%	0.64%	5.54%	0.55%	7.94%
$ \hat{\pi} - \pi  > 0.5 * \hat{\pi}$	1.78%	17.32%	1.59%	10.93%	0.75%	14.73%
$ \hat{\pi} - \pi  > 0.75 * \hat{\pi}$	2.16%	23.42%	2.36%	15.53%	0.89%	21.64%
$ \hat{\pi} - \pi  > 1 * \hat{\pi}$	1.97%	25.65%	3.06%	20.25%	1.99%	26.29%
$ \hat{\pi} - \pi  > 2 * \hat{\pi}$	2.99%	26.77%	4.58%	23.38%	5.28%	28.34%
$ \hat{\pi} - \pi  > 3 * \hat{\pi}$	4.83%	24.50%	3.88%	21.80%	4.73%	29.00%
$ \hat{\pi} - \pi  > 4 * \hat{\pi}$	4.52%	23.59%	1.53%	19.71%	3.50%	28.13%
$ \hat{\pi} - \pi  > 5 * \hat{\pi}$	4.13%	22.14%	2.10%	16.81%	1.51%	25.73%
$ \hat{\pi} - \pi  > 7.5 * \hat{\pi}$	1.59%	17.80%	0.32%	11.40%	0.62%	19.47%
$ \hat{\pi} - \pi  > 10 * \hat{\pi}$	0.19%	12.57%	-0.83%	7.22%	-0.07%	14.18%

**Table 10: Constrained - Three Factor: (1/2 sales, 1/4 PPE, 1/4 labor measure)**

Sample <sup>1</sup>	Operating Income		Taxable Income Derived from Taxes		Market Capitalization	
	1	2	1	2	1	2
N (Sample) <sup>2</sup>	1575	6992	1574	6986	1461	6384
N (0-value for $\hat{\pi}$ ) <sup>3, 4</sup>	3	24	3	24	2	21
N (mean computation) <sup>5</sup>	1572	6968	1571	6962	1459	6363
mean[ $ \hat{\pi} - \pi  / \hat{\pi}$ ]	41.63	24.55	1.62	5.04	10.27	39.72
median[ $ \hat{\pi} - \pi  / \hat{\pi}$ ]	2.02	2.35	1.00	1.00	1.55	2.59
Percent of firms for which:						
$ \hat{\pi} - \pi  > 0.05 * \hat{\pi}$	97.84%	98.09%	98.15%	98.43%	98.08%	98.15%
$ \hat{\pi} - \pi  > 0.1 * \hat{\pi}$	96.31%	96.66%	95.93%	96.94%	96.98%	96.54%
$ \hat{\pi} - \pi  > 0.2 * \hat{\pi}$	92.56%	93.50%	91.73%	93.94%	92.25%	93.59%
$ \hat{\pi} - \pi  > 0.25 * \hat{\pi}$	90.90%	91.73%	89.62%	92.63%	89.72%	91.83%
$ \hat{\pi} - \pi  > 0.5 * \hat{\pi}$	83.14%	83.74%	80.52%	84.96%	80.40%	82.51%
$ \hat{\pi} - \pi  > 0.75 * \hat{\pi}$	75.06%	76.98%	70.27%	77.54%	70.32%	73.90%
$ \hat{\pi} - \pi  > 1 * \hat{\pi}$	68.38%	70.62%	46.47%	45.73%	61.00%	66.59%
$ \hat{\pi} - \pi  > 2 * \hat{\pi}$	50.38%	54.06%	24.00%	30.88%	39.48%	55.18%
$ \hat{\pi} - \pi  > 3 * \hat{\pi}$	31.23%	44.35%	12.22%	23.14%	23.58%	46.31%
$ \hat{\pi} - \pi  > 4 * \hat{\pi}$	18.45%	37.74%	6.87%	17.44%	14.80%	39.78%
$ \hat{\pi} - \pi  > 5 * \hat{\pi}$	13.04%	32.82%	3.95%	13.29%	11.10%	33.96%
$ \hat{\pi} - \pi  > 7.5 * \hat{\pi}$	8.40%	23.48%	1.15%	7.07%	6.79%	23.59%
$ \hat{\pi} - \pi  > 10 * \hat{\pi}$	6.87%	18.10%	0.64%	3.96%	5.00%	18.39%
vs. unconstrained model <sup>6</sup>	<i>(percentage point difference)</i>					
$ \hat{\pi} - \pi  > 0.05 * \hat{\pi}$	-0.64%	1.13%	0.38%	0.76%	-0.34%	0.99%
$ \hat{\pi} - \pi  > 0.1 * \hat{\pi}$	0.06%	2.65%	0.06%	1.29%	0.55%	2.48%
$ \hat{\pi} - \pi  > 0.2 * \hat{\pi}$	-0.32%	5.41%	-0.06%	2.82%	-0.27%	4.89%
$ \hat{\pi} - \pi  > 0.25 * \hat{\pi}$	0.51%	6.33%	-0.25%	3.88%	-0.41%	6.21%
$ \hat{\pi} - \pi  > 0.5 * \hat{\pi}$	1.08%	12.26%	0.57%	7.14%	0.00%	10.20%
$ \hat{\pi} - \pi  > 0.75 * \hat{\pi}$	0.64%	16.91%	0.25%	10.70%	-0.69%	14.47%
$ \hat{\pi} - \pi  > 1 * \hat{\pi}$	-0.25%	17.55%	-0.51%	13.46%	-0.48%	17.41%
$ \hat{\pi} - \pi  > 2 * \hat{\pi}$	-1.08%	16.89%	-0.76%	15.23%	0.55%	19.28%
$ \hat{\pi} - \pi  > 3 * \hat{\pi}$	-0.57%	15.18%	-0.38%	13.73%	0.82%	19.05%
$ \hat{\pi} - \pi  > 4 * \hat{\pi}$	0.45%	14.61%	-0.83%	11.53%	0.48%	17.87%
$ \hat{\pi} - \pi  > 5 * \hat{\pi}$	0.83%	13.58%	-0.32%	8.83%	0.21%	15.20%
$ \hat{\pi} - \pi  > 7.5 * \hat{\pi}$	0.32%	8.63%	-1.08%	4.60%	-0.21%	9.34%
$ \hat{\pi} - \pi  > 10 * \hat{\pi}$	-0.38%	5.15%	-1.40%	2.03%	-0.48%	6.40%

**Table 6**  
**Income Prediction Accuracy among Formulas with Labor Compensation**

Notes: The Table presents mean square errors from regressions explaining 2004 profits as functions of contemporaneous sales, capital, and labor costs. All equations are estimated using weighted least squares, with weights equal to one divided by the square root of contemporaneous firm assets.

<i>Dependent Variable:</i>	Operating Income			Taxable Income Reflected in Taxes Paid			Market Capitalization	
	Obs.	MSE	% higher MSE	Obs.	MSE	% higher MSE	Obs.	MSE
3-factor, unconstrained	1575	28,068,291	0.00%	1574	18,631,321	0.00%	1461	2.609E-
3-factor, equal weights	1575	33,499,441	19.35%	1574	24,315,983	30.51%	1461	2.830E-
3-factor, double sales weights	1575	31,173,686	11.06%	1574	22,162,656	18.95%	1461	2.688E-
2-factor (sales and PPE), unconstrained	1575	28,058,319	-0.04%	1574	18,692,620	0.33%	1461	2.684E-
2-factor (sales and PPE), constrained	1575	29,226,697	4.13%	1574	20,250,335	8.69%	1461	2.810E-
2-factor (sales and wages), unconstrained	1575	29,590,430	5.42%	1574	19,198,054	3.04%	1461	2.717E-
2-factor (sales and wages), constrained	1575	39,674,233	41.35%	1574	28,568,962	53.34%	1461	3.258E-
2-factor (PPE and wages), unconstrained	1575	34,695,905	23.61%	1574	25,438,003	36.53%	1461	3.073E-
2-factor (PPE and wages), constrained	1575	39,027,225	39.04%	1574	28,922,762	55.24%	1461	3.234E-
1-factor (sales)	1575	29,583,480	5.40%	1574	19,252,446	3.33%	1461	2.793E-
1-factor (PPE)	1575	36,048,545	28.43%	1574	26,057,458	39.86%	1461	3.422E-
1-factor (wages)	1575	51,415,773	83.18%	1574	37,441,279	100.96%	1461	4.243E-



## Application to tax policy

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- We can use simple econometrics to estimate the reliability of alternative factor weights for formula apportionment.
- Commonly proposed factors explain income rather poorly.
- Labor factors do a particularly bad job, which is probably not surprising.
- The inaccuracy of these predictions implies that there would be significant deadweight losses from tax effects on ownership by adopting formula apportionment internationally.
- Does this imply that formula apportionment is always a bad idea, or that employment factors should never be used?
- Not necessarily, since the policy choice depends on the alternatives. But this analysis does suggest that there are significant costs from using formulas, even with optimally chosen factors.